Market Timing with Moving Averages: The Anatomy and Performance of Trading Rules

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I dedicate this book to all technical traders, especially newcomers to the market, and hope that it helps them better understand the tools at their disposal.
PREFACE

About the Author

Valeriy Zakamulin is Professor of Finance at the School of Business and Law, University of Agder, Norway, where he teaches graduate courses in Finance. His first graduate academic degree is a MS in Radio Engineering. After receiving this degree, Valeriy Zakamulin had been working for many years as a research fellow at a computer science department, developing both computer hardware and software. Later on Valeriy Zakamulin received a MS in Economics and Business Administration and a PhD in Finance. He has published more than 30 articles in various refereed academic and practitioner journals and is a frequent speaker at international conferences. He has also served on editorial boards of several economics and finance journals. His current research interests cover behavioral finance, portfolio optimization, time-series analysis of financial data, financial asset return and risk predictability, and technical analysis of financial markets.
Motivation for Writing this Book

Over the course of the last decade, the author of this book has been interested in the stock return predictability which is one of the most controversial topics in financial research. The existence of stock return predictability is of great interest to both practitioners and academics alike. Traditionally, in finance literature the stock returns were predicted using various financial ratios and macroeconomic variables. Unfortunately, the evidence of stock return predictability by either financial ratios or macroeconomic variables is unconvincing. Technical analysis represents another methodology of predicting future stock returns through the study of past stock prices and uncovering some recurrent regularities, or patterns, in price dynamics.

Whereas technical analysis has been extensively used by traders for almost a century and the majority of active traders strongly believe in stock return predictability, academics had long been skeptical about the usefulness of technical analysis. Yet the academics’ attitude towards the technical analysis is gradually changing. The findings in a series of papers on technical analysis of financial markets suggest that one should not bluntly dismiss the value of technical analysis. Recently we have witnessed a constantly increasing interest in technical analysis from both practitioners and academics alike. This interest developed because over the decade of 2000s, that covers two severe stock market downturns, many technical trading rules outperformed the market by a large margin.

One of the basic principles of technical analysis is that “prices move in trends”. Traders firmly believe that these trends can be identified in a timely manner and used to generate profits and limit losses. Consequently, trend following is the most widespread trading strategy; it tries to jump on a trend and ride it. Specifically, when stock prices are trending upward (downward), it’s time to buy (sell) the stock. The problem is that stock prices fluctuate wildly which makes it difficult for traders to identify the trend in stock prices. Moving averages are used to “smooth” the fluctuations in the stock price in order to highlight the underlying trend. As a matter of fact, a moving average is one of the oldest and most
popular tools used in technical analysis for detecting a trend.

Over the course of the last few years, the author of this book has conducted research on the profitability of moving average trading rules. The outcome of this research was a collection of papers, two of which were published in scientific journals. The rest of the papers in this collection laid the foundations for this book on market timing with moving averages.

In principle, there are already many books on technical analysis of financial markets that cover the subject of trading with moving averages. Why a new book on moving averages? The reasons for writing a new book are explained below.

All existing books on trading with moving averages can be divided into two broad categories:

1. Books that cover all existing methods, tools, and techniques used in technical analysis of financial markets (two examples of such books are Murphy, 1999, and Kirkpatrick and Dahlquist, 2010). In these books, that can be called as the “Bibles” of technical analysis, the topic on technical trading with moving averages is covered briefly and superficially; the authors give only the most essential information about moving averages and technical trading rules based on moving averages.

2. Books that are devoted solely to the subject of moving averages (examples of such books are Burns and Burns, 2015, and Droke, 2001). These books are usually written for beginners; the authors cover in all details only the most basic types of moving averages and technical trading rules based on moving averages.

Regardless of the book type, since the subject of technical trading with moving averages is constantly developing, the information in the existing books is usually outdated and/or obsolete. Thereby the existing books lack in-depth, comprehensive, and up-to-date information on technical trading with moving averages.

Unfortunately, the absence of a comprehensive handbook on technical trading with moving averages is just one of several issues with the subject. The other two important issues are as follows:
1. There are many types of moving averages as well as there are many technical trading rules based on one or several moving averages. As a result, technical traders are overwhelmed by the variety of choices between different types of trading rules and moving averages. One of the controversies about market timing with moving averages is over which trading rule in combination with which moving average(s) produces the best performance. The situation is further complicated because in order to compute a moving average one must specify the size of the averaging window. Again, there is a big controversy over the optimal size of this window for each trading rule, moving average, and financial market. The development in this field has consisted in proposing new ad-hoc rules and using more elaborate types of moving averages in the existing rules without any deeper analysis of commonalities and differences between miscellaneous choices for trading rules and moving averages. It would be no exaggeration to say that the existing situation resembles total chaos and mess from the perspective of a newcomer to this field.

2. Virtually all existing books and the majority of papers on technical trading with moving averages claim that one can easily beat the market and become rich by using moving averages. For example, in one popular paper the author claims that using moving averages in the stock market produces “equity-like returns with bond-like volatility and drawdowns” (that is, moving averages produce stock-like returns with bond-like risk). There are many similar claims about the allegedly superior performance of moving average trading strategies. The major problem is that all these claims are usually supported by colorful narratives and anecdotal evidence rather than objective scientific evidence. At best, such claims are “supported” by performing a simple back-test using an arbitrary and short historical sample of data and reporting the highest observed performance of a trading rule. Yet, serious researchers know very well that the observed performance of the best trading rule in a back test severely overestimates its real-life performance.
Overall, despite a series of publications in academic journals, modern technical analysis in general and trading with moving averages in particular still remain art rather than science. In the absence of in-depth analysis of commonalities and differences between various trading rules and moving averages, technical traders do not really understand the response characteristics of the trading indicators they use and the selection of a specific trading rule, coupled with some specific type of a moving average, is made based mainly on intuition and anecdotal evidence. Besides, there is usually no objective scientific evidence which supports the claim that some specific moving average trading strategy allows one to beat the market.

To the best knowledge of the author, there is only one book to date (Aronson, 2010), that conveys the idea that all claims in technical analysis represent, in principle, scientific testable claims. The book describes carefully all common pitfalls in back-testing trading rules and presents correct scientific methods of testing the profitability of technical trading rules. The book contains a thorough review of statistical principles with a brief case study of profitability of various technical trading rules (including a few moving average trading rules) in one specific financial market. Therefore, whereas the book makes a very good job in explaining how to scientifically evaluate the performance of trading rules, the case study in the book is very limited; the question of how profitable are the moving average trading rules in various financial markets remains unanswered.

**Book Objectives and Structure**

Given the increasing popularity of trading with moving averages, we thought of writing this book in order to overcome the shortcomings of existing books and give the readers the most comprehensive and objective information about this topic. Specifically, the goals of this book are threefold:

1. Provide the in-depth coverage of various types of moving averages, their properties, and technical trading rules based on moving averages.

2. Uncover the anatomy of market timing rules with moving averages and offer a new
and very insightful reinterpretation of the existing rules.

3. Revisit the myths regarding the superior performance of moving average trading rules and provide the reader with the most objective assessment of the profitability of these rules in different financial markets.

This book is composed of four parts and a concluding chapter; each part consists of two or three chapters:

**Part I:** This part provides the in-depth coverage of various types of moving averages and their properties.

**Chapter 1:** This chapter presents a brief motivation for using moving averages for trend detection, how moving averages are computed, and their two key properties: the average lag (delay) time and smoothness. The most important thing to understand right from the start is that there is a direct relationship between the average lag time and smoothness of a moving average.

**Chapter 2:** This chapter introduces the notion of a general weighted moving average and shows that each specific moving average can be uniquely characterized by either a price weighting function or a price-change weighting function. It also demonstrates how to quantitatively assess the average lag time and smoothness of a moving average. Finally, the analysis provided in this chapter reveals two important properties of moving averages when prices trend steadily.

**Chapter 3:** This chapter presents a detailed review of all ordinary types of moving averages, as well as some exotic types of moving averages. These exotic moving averages include moving averages of moving averages and mixed moving averages with less average lag time. For the majority of moving averages, this chapter computes the closed-form solutions for the average lag time and smoothness. This chapter also demonstrates that the average lag time of a moving average can easily be manipulated; therefore the notion of the average lag time has very
little to do with the delay time in the identification of turning points in a price trend.

Part II: This part reviews the technical trading rules based on moving averages and uncovers the anatomy of these rules.

Chapter 4: This chapter reviews the most common trend-following rules that are based on moving averages of prices. It also discusses the principles behind the generation of trading signals in these rules. This chapter also illustrates the limitations of these rules and argues that the moving average trading rules are advantageous only when the trend is strong and long-lasting.

Chapter 5: This key chapter presents a methodology for examining how the trading signal in a moving average rule is computed. Then using this methodology the chapter examines the computation of trading signals in all moving average rules and investigates the commonalities and differences between the rules. The main conclusion that can be drawn from this study is that the computation of the trading indicator in every rule, based on either one or multiple moving averages, can equivalently be interpreted as the computation of a single weighted moving average of price changes. The analysis presented in this chapter uncovers the anatomy of moving average trading rules, provides very useful insights about popular trend rules, and offers a new reinterpretation of the existing moving average trading rules.

Part III: In this part we present our methodology for how to scientifically test the claim that one can beat the market by using moving average trading rules.

Chapter 6: This chapter starts with a review of transaction costs in capital markets. Then it demonstrates how to simulate the returns to a moving average trading strategy in the presence of transaction costs. The following two cases are considered when a trading indicator generates a sell signal: case one where the
trader switches to cash, and case two where the trader alternatively sells short a financial asset.

Chapter 7: This chapter explains how to evaluate the performance of a trading strategy and how to carry out a statistical test of the hypothesis that a moving average trading strategy outperforms the corresponding buy-and-hold strategy. In particular, it argues that there is no unique performance measure, reviews the most popular performance measures, and points to the limitations of these measures. The chapter then surveys the parametric methods of testing the outperformance hypothesis and the current “state of the art” non-parametric methods.

Chapter 8: Technical traders typically rely on back-testing which is defined as the process of testing a trading strategy using relevant historical data. Back-testing usually involves “data mining” which denotes the practice of finding a profitable trading strategy by extensive search through a vast number of alternative strategies. This chapter explains that the data-mining procedure tends to find a strategy which performance benefited most from luck. As a result, the performance of the best strategy in a back test is upward biased. This fact motivates that any back-test must be combined with a data-mining correction procedure that adjusts downward the estimated performance. Another straightforward method of the estimation of true performance of a trading strategy is to employ a validation procedure; this method is called forward-testing.

Part IV: This part contains case studies of profitability of moving average trading rules in different financial markets.

Chapter 9: This chapter utilizes the longest historical sample of data on the S&P Composite stock index and comprehensively evaluates the profitability of various moving average trading rules. Among other things, the chapter investigates the following: which trading rules performed best; whether the choice of moving average influences the performance of trading rules; how accurately the trading
rules identify the bullish and bearish stock market trends; whether there is any advantage in trading daily rather than monthly; and how persistent is the outperformance delivered by the moving average trading rules. The results of this study allow us to revisit the myths regarding the superior performance of the moving average trading rules in this well-known stock market and fully understand their advantages and disadvantages.

Chapter 10: This chapter tests the profitability of various moving average trading rules in different financial markets: stocks, bonds, currencies, and commodities. The results of these tests allow us to better understand the properties of the moving average trading strategies and find out which trading rules are profitable in which markets. The chapter concludes with a few practical recommendations for traders testing the profitability of moving average trading rules. The analysis presented in this chapter also suggests a hypothesis about simultaneous existence, in the same financial market, of several trends with different durations.

Conclusion, Chapter 11: This concluding chapter presents a brief summary of the key contributions of this book to the field of technical analysis of financial markets. In addition, the chapter derives an alternative representation of the main result on the anatomy of moving average trading rules. It is demonstrated that all these rules predict the future price trend using a simple linear forecasting model that is identical to models used in modern empirical finance. Therefore this alternative representation allows us to reconcile modern empirical finance with technical analysis of financial markets that uses moving averages. Finally, this chapter discusses whether the advantages of the moving average rules, observed using past (historical) data, are likely to persist in the future.
Readership and Prerequisites

This book is not for a layman who believes that moving averages offer a simple, quick, and easy way to riches. This book is primarily intended for a serious and mathematically minded reader who wants to get in-depth knowledge of the subject. Even though, for the sake of completeness of exposition, we briefly cover all relevant theoretical topics, we do not explain the basic financial terminology, notions, and jargons. Therefore this book is best suited for the reader with a MS degree in economics or business administration who is familiar with basic concepts in investments and statistics. Examples of such readers are academics, students at economic departments, and practitioners (portfolio managers, quants, traders, etc.). This book is, in principle, also suited for self-study by strongly motivated readers without prior exposure to finance theory, but in this case the book should be supplemented by an introductory textbook on investments at least (an example of such book is Bodie, Kane, and Marcus, 2007).

Parts I and II are relatively easy to comprehend. These parts require only the knowledge of high school mathematics, basically a familiarity with arithmetic and geometric series and their sums. The material presented in Parts III and IV of this book makes it necessary to use extensively financial mathematics and statistics. Without the required prerequisites, the reader can try to skip Part III of the book and jump directly to Part IV. However, in order to understand the results reported in Part IV of this book, the reader is required to have a superficial knowledge of back-tests and forward-tests, and to understand our notion of “outperformance” which is the difference between the performances of the moving average trading strategy and the corresponding buy-and-hold strategy.

Supplementary Book Materials

The author of this book provides two types of supplementary book materials that are available online on the author’s website http://vzakamulin.weebly.com/.

The first type of supplementary book materials is interactive web applications. Interac-
tivity means that outputs in these applications change instantly as the user modifies inputs. Therefore these applications not only replicate the illustrations and results provided in this book, but also allow the user to modify inputs and get new illustrations and results. Last but not least, these applications offer the user real-time trading signals for some stock market indices. There are no any prerequisites for using the first type of supplementary book materials.

The results reported in this book were obtained using the open source programming language R (see https://www.r-project.org/). To let anyone reproduce some of the results provided in this book, as well as test the profitability of moving average trading rules using own data, the author provides the second type of supplementary book materials: two R packages that include reusable R functions, the documentation that describes how to use them, and sample data. The first R package is bbDetection that allows the user to detect bull and bear states in a financial market and to get the dating and the descriptive statistics of these states. The second R package is matiming that allows the user to simulate the returns to different moving average trading rules and to perform both back-tests and forward-tests of the trading rules. The prerequisites for using the second type of supplementary book materials are the familiarity with R language and the ability to write R programs.